which it commenced, *i.e.*, perpendicular to the axis; but when the steel is soft, the plane of the tear gradually tilts over and coincides with the surface of least resistance to shearing, *i.e.*, becomes inclined at 45° to the axis.

Now, at rupture, an originally soft bar is harder in the centre of the narrowed section than at the circumference, where the drawing out has been less; hence, fracture commences at the centre perpendicular to the axis, and tears outwards until it reaches the softer material, when it will continue along a surface of least resistance to shearing, *i.e.*, along a surface formed by the intersection of two cones. Hence, we find the fracture of a soft steel bar consisting of a crater with a more or less extended base; see figs. 5 and 6, Plate 2, and 6 and 7, Plate 3.

The harder the steel, at the outset, the broader will be the base of the crater, until, in very hard steels, there is only a rim or crown left round the edge; and in the hardest steels all trace of the surface of least resistance to shearing disappears.

[*Note.*—I have employed the term "hard" in the sense usually understood, *i.e.*, where the "hardness" is measured by the value of the limit of elastic resistance.]

"Photometric Observations of the Sun and Sky." By WILLIAM BRENNAND. Communicated by C. B. CLARKE, F.R.S.

Received October 30,—Read December 11, 1890.

1. In the publications of the Society from 1859 to 1870, many communications by Sir Henry Roscoe on this subject will be found. Of these, the most important bearing directly on my observations are—

a. Bunsen and Roscoe, "On the Direct Measurement of the Chemical Action of Sunlight," in 'Phil. Trans.,' 1863, pp. 139–160.

It is proved, *inter alia*, that equal shades are produced in photographically sensitised paper by equal products of intensity of light × time of insolation. The preparation of a photographic paper which shall always possess the same degree of sensitiveness is carefully described.

b. Roscoe, "On a Method of Meteorological Registration of the Chemical Action of Total Daylight," in 'Phil. Trans.,' 1865, pp. 605–631 [Bakerian Lecture].

The law is stated, *inter alia*, that light of intensity 50 acting for 1 second has the same effect as light of intensity 1 acting for 50 seconds.

The mechanical arrangement for exposing the paper horizontal, or by the aid of a vertical drum, is explained.

Tables are added of half-hourly readings at Manchester, giving general actinic effects for different seasons of the year, &c.

c. Roscoe and Baxendell, "On the Relative Chemical Intensities of direct Sunlight and diffuse Daylight at different Altitudes of the Sun," in 'Roy. Soc. Proc.,' vol. 15, 1866–67, pp. 20–24.

By "total daylight" is meant the whole resultant action of the Sun and sky on paper exposed horizontally.

By "diffuse daylight" is meant the same action when the Sun was stopped out.

The "direct sunlight" was taken as the difference between these two; it does not appear to have been observed directly.

d. Roscoe, "On the Chemical Intensity of Total Daylight at Kew and Pará," in 'Phil. Trans.,' 1867, pp. 555–570.

e. Roscoe and Thorpe, "On the Relation between the Sun's Altitude and the Chemical Intensity of Total Daylight in a Cloudless Sky," in 'Phil. Trans.', 1870, pp. 309–316.

2. My observations made at Dacca, in 1861–1866 (repeated at Milverton, in Somersetshire, during the last year), were made in entire ignorance of the work of Sir H. Roscoe; his results, therefore, so far as they agree with mine, afford an independent support to my theory. My experiments have been directed largely to ascertaining the laws of the distribution of the actinic power in the sky, and thus the work of Sir H. Roscoe overlaps mine at particular points only. So also Roscoe has taken numerous observations of the sky more or less clouded; I take no observation except when the sky is clear, as I find even a very slight haze to produce large differences in the measurements, and to bring into the numerical results complications that I have not at present attempted to deal with.

3. The method of measurement I adopted, is the darkening produced in sensitised photographic paper; for this effect I accept Roscoe's term of "the chemical action." My method of measurement differs from that of Roscoe in one important point: I use strips cut from one uniform sheet of ordinary photographic paper; all my measurements are so far relative, and I obtain the same numerical results (ratios) with any paper. I compare ultimately the effect of the Sun and of a candle on this same paper. Roscoe, by preparing special paper with definite proportions of nitrate of silver, &c., depends on thus reproducing paper of exactly the same sensitiveness. I make each measurement numerically (as did Roscoe) by comparing the shade produced with some standard blackness.

4. I assume that in the burning of a stearine candle, the "chemical action" is proportional to the material consumed. I have taken as my unit (i) of measure of chemical action, the darkening produced at a distance of 1 inch from the wick of the candle, when 100 grains were consumed, which, in the candle I used in India, occupied about

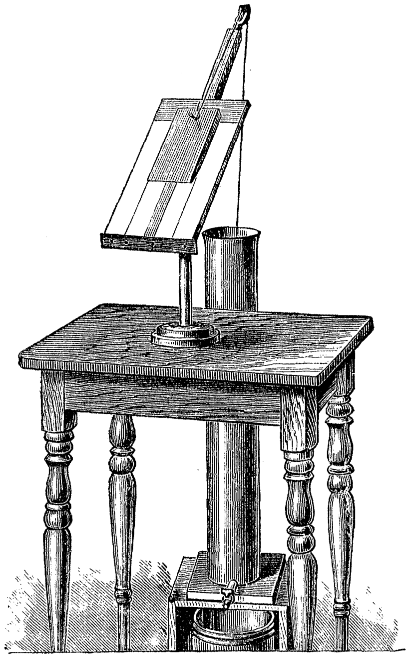
47 minutes. [I am here narrating the course I pursued in commencing these observations at Dacca; I very soon discarded the candle, as I was able, by the aid of my table given below, to recover the unit of measurement by a Sun observation.]

5. I form a strip of photographic paper about $\frac{3}{4}$ inch deep into a circular ring, placed inside a metal cylinder 3 inches in diameter. I place now my standard candle eccentrically, at a distance of 1 inch from the surface, and burn the 100 grains of stearine. I thus get a strip which is gradually coloured from the point nearest the centre (where the intensity is unit i) to the most remote point (where the intensity is $\frac{1}{4}$ i). By calculating the distances of various points of the ring from the wick, the intensities corresponding to these distances can be marked. I exhibit a small strip (of somewhat different dimensions) so calibrated to show a scale of intensities; it has lost its original shade in consequence of fixing and toning. For actual purposes of measurement, a strip is used in its original unfixed state.

6. My earlier observations on the chemical action of the Sun and sky, were made in Bengal, with a "mica actinometer." In this, small squares of one sheet of sensitised paper were covered by 1, 2, 3, 4 . . . . thicknesses of mica cut from the same plate; the sheet of paper then exposed to any light for a certain time gave me a series of chequered shades. To measure the effect of the Sun or of any portion of the sky, I noted the time necessary to darken the paper till it matched one of the squares in blackness. This instrument I have long since laid aside, as I have superseded it by better; but by its aid in 1863 I was led to the attempt of measuring the chemical action of the Sun, in a clear sky, for each degree of the Sun's altitude, so as to form a table of constants, which would render a direct reference to the candle power unnecessary.

7. I have made an instrument (fig. 1) similar to one employed in India. The plane on which to expose the sensitised paper has motions in altitude and azimuth; a perpendicular style is placed at the corner; and, by shifting the plane until the style casts no shadow, the plane can be adjusted at right angles to the Sun's rays, and the Sun's altitude can be read by a brass Gunter's quadrant. A slide which covers the strip of sensitised paper, is made to move uniformly up the plane, by means of a string passing over a pulley attached to a float in a column of water in a long cylinder (the one used in India was a rain-gauge); the float descends as the water is drawn off by a stopcock at the bottom of the cylinder. Lines can be drawn on a gauge pasted on the plane, beside the longitudinal slit, in which is exposed the sensitised paper, corresponding to the motion for 1, 2, 3, ... 20 seconds; also a second gauge has been drawn for a larger tap giving quicker motion. By simply moving the sensitised paper

Fig. 1.



laterally, a fresh portion of it is brought under the longitudinal slit, and the observation can be immediately repeated, several times if so desired.

8. By comparing the darkening produced in the paper in paragraphs 5 and 7, we easily show that we have to expose the paper four times as long to produce the effect caused by diminishing the distance one half; and that a light of intensity 4 acting for 1 second has the same effect as a light of intensity 1 acting for 4 seconds. This I think might have been assumed; Bunsen and Roscoe, in their paper (1863) above cited, have, however, taken great pains to prove it.

9. My early experiments were designed to test the total effect of the sky and Sun for photographic purposes. I have always experimented mainly by exposing the paper at right angles to the Sun's rays. Roscoe on the other hand, exposes his paper in a horizontal plane. It will be seen below, that theoretic considerations have led me to another method of observation, which gives directly the measure of effect really desired, and does not require a clear heavens down to the horizon on all sides (the Octant Actinometer). I give as a first example of my experiments the following table (A). The observation

was taken on 21st December, 1863, on the roof of my house at Dacca, the sky being perfectly clear. The paper was exposed at right angles to the Sun, thus giving the effect of the Sun, together with the total effect (resolved on the plane at right angles to the Sun) of that portion of the visible sky within 90° of the Sun.

Table A.

Sun's altitude.	Time of observation.	Number of seconds per inch in motion of slide.	Length in inches of strip for constant shading, C.	Chemical action measured in unit I.
11° 0′	7^h 41^m A.M.	11·0	1·52	0·06
14 0	8 2	12·0	1·2	0·07
19 0	8 31	6·2	1·8	0·088
24 46	9 17	6·0	1·26	0·132
29 0	9 33	7·25	0 74	0·186
32 0	9 51	6·7	0·77	0·192
34 30	10 12	6·0	0·82	0·203
36 48	10 26	6·7	0·68	0·219
39 0	10 54	6·5	0·683	0·226
41 30	11 22	6·5	0·625	0·24
41 40	11 35	6·7	0·59	0·2525
42 20	11 50	6·7	0·56	0·266
42 30	12 0	6·25	0·6	0·269

The number in the fifth column in this table, is the reciprocal of the product of the two numbers in the third and fourth columns.

Thus, taking the last but one observation,

$$6{\cdot}7 \times 0{\cdot}56 = 3{\cdot}752,$$

and

$$\frac{1}{3{\cdot}752} = 0{\cdot}266.$$

The constant C of shading used as the standard of comparison was the tint produced in the same paper by the candle burning 47 minutes at 1 inch distance. Hence, the unit I here employed was 47×60 times i the unit in paragraph 4 above.

In order to get a deeper shade of darkening in the first two observations, when the Sun was low, a smaller stopcock was used than in the succeeding observations.

In each of these observations, the actual velocity of the slide was observed by an assistant with a watch. As explained in paragraph 7, this constant can be obtained more easily and exactly by a gauge, pasted on the plane beside the slit, graduated for the stopcock used.

10. The observations of Table (A), and numerous other similar observations, were taken with great care, the strips being read the same evening. The strips taken on separate days, were also compared with each other; it was thus found that the numerical values for the chemical action were the same, with different paper, and with different candles. In England, I have, within the last two years, made similar observations to those at Dacca twenty-five years ago, and I submit three of the strips taken; these have been fixed, and have consequently changed both in density and in colour, and are submitted merely for explanation. The photographic sensitised paper, now prepared in England, keeps in the dark for months unchanged, and renders constant reference to the candle standard unnecessary. But by the aid of the table (B) which immediately follows, I could always in 1889 and 1890 recover the standard unit, by an observation of the Sun better than from the candle.

11. The chemical action of the Sun alone, is got in a dark room, by arranging a vertical slit, so that the Sun's light falls exactly down the strip of paper, which I expose at right angles to his rays. To get the chemical action of the Sun and sky (*i.e.*, the portion of the visible sky within 90° of the Sun) together, the exposure is completely in the open. The chemical action of the sky (*i.e.*, the resultant action on the plane at right angles to the Sun of that portion of the visible sky within 90° of the Sun) is got by an exposure in the open, a vertical stick having been arranged so that its shadow should just cover the exposed strip.

As I took each of these three kinds of observations, giving numerical results α, β, γ respectively, I was enabled from the simple formula $\alpha + \gamma = \beta$ to check my observations, to test the closeness with which the strips could be read certainly, and to show again that an intensity of 4 acting for 1 second has the same effect as the intensity of 1 acting for 4 seconds.

12. I found, as Roscoe, working in a less pure atmosphere, found in a still greater degree, that observations very close to the horizon were not to be depended upon. Also, in the cold weather at Dacca, at which season alone the sky was sufficiently clear, the Sun did not attain a greater altitude than about 45°. The flat roof of my house offered nearly a complete hemisphere of unclouded blue; nevertheless, I know that the full effect of the band of sky near the horizon, must have been to some extent interfered with by haze; and the constants in some of the tables that follow will be, in a small degree, affected by this cause.

13. The following Table (B) is shortened, from one which I drew up and printed photographically at Dacca in 1865. It represents the mean result of very numerous observations, taken at altitudes of the Sun between 5° and 45°. From 45° to 90°, the table has been

partially completed by using the formula $i = 0{\cdot}494\ (0{\cdot}991)^{\epsilon}$ where ϵ is the distance traversed by the Sun's rays in the atmosphere for different altitudes. This formula is parallel to the equation $t = Ap^{\epsilon}$ used by Pouillet in his memoir on the Solar Heat (and can be found in 'Taylor's Memoirs,' vol. 4, p. 49).

N.B.—In this table, in each observation the sensitised paper was exposed at right angles to the Sun's rays: so that a different portion of the sky was observed at each altitude.

Table B.

Chemical Action of Sun and Sky.

Sun's altitude.	Sun alone.	Sky alone.	Sun and sky together.	Sun's altitude.	Sun alone.	Sky alone.	Sun and sky together.
5°...	0·0064	0·0125	0·0189	31°...	0·1120	0·0636	0·1739
6...	0·0090	0·0156	0·0246	32...	0·1134	0·0643	0·1777
7...	0·0120	0·0189	0·0390	33...	0·1158	0·0648	0·1806
8...	0·0156	0·0224	0·0380	34...	0·1185	0·0654	0·1839
9...	0·0196	0·0256	0·0453	35...	0·1215	0·0660	0·1875
10...	0·0238	0·0288	0·0526	36...	0·1238	0·0665	0·1903
11...	0·0283	0·0319	0·0602	37...	0·1262	0·0670	0·1932
12...	0·0330	0·0349	0·0679	38...	0·1290	0·0675	0·1965
13...	0·0377	0·0376	0·0782	39...	0·1307	0·0678	0·1985
14...	0·0423	0·0401	0·0824	40...	0·1331	0·0683	0·2014
15...	0·0471	0·0425	0·0896	41...	0·1349	0·0686	0·2035
16...	0·0519	0·0447	0·0966	42...	0·1369	0·0689	0·2058
17...	0·0566	0·0477	0·1043	43...	0·1384	0·0692	0·2076
18...	0·0612	0·0486	0·1098	44...	0·1407	0·0695	0·2103
19...	0·0657	0·0594	0·1161	45...	0·1429	0·0700	0·2128
20...	0·0701	0·0520	0·1221				
21...	0·0750	0·0529	0·1279				
22...	0·0786	0·0549	0·1335	50...	0·1504	0·0711	0·2215
23...	0·0826	0·0562	0·1388	55...	0·1568	0·0720	0·2288
24...	0·0865	0·0574	0·1439	60...	0·1620	0·0727	0·2347
25...	0·0905	0·0583	0·1488	65...	0·1662	0·0732	0·2394
26...	0·0939	0·0595	0·1534	70...	0·1696	0·0736	0·2432
27...	0·0974	0·0604	0·1578	75...	0·1721	0·0739	0·2461
28...	0·1008	0·0613	0·1621	80...	0·1737	0·0741	0·2478
29...	0·1040	0·0621	0·1661	85...	0·1747	0·0742	0·2489
30...	0·1070	0·0628	0·1698	90...	0·1751	0·0743	9·2494

14. It must be carefully noted, with respect to these older observations, that what I actually observed, was the number of seconds' exposure at each altitude necessary to produce a particular darkening in the sensitised paper, viz., the shade produced by the constant candle at distance unity, in that particular paper; the numbers printed

were obtained by taking the inverse of these times for the chemical action.

15. The table is only a first approximation; yet I have much greater confidence in the values, than in those given by any one observation, the table itself being deduced from a very large number of experiments.

Sir H. Roscoe believes (Bakerian Lecture, 1865) that he brought the errors due to matching shades to within 2 per cent. correct; and in graduating strips, the mean error was found by him not to exceed 1 per cent. of the measured intensity. I am not satisfied that my separate observations were always so closely accurate in the matching of shades. I employed my daughters independently, to match shades, and compared them with my own reading, and found that the readings *sometimes* differed more than 2 per cent.

The photographic paper employed, varied somewhat in tint; that exposed to the candle being a little redder than that exposed to the Sun and sky; the same intensity in the darkening was sought in every case. I suppose the difference in tint to have been due to the heat of the candle.

16. The effect of the sky observed, was that due to the effect of each elemental area of it multiplied by the sine of the angle between that elemental area and the normal to the plane of exposure, these infinitesimal effects being summed throughout the visible sky within 90° of the Sun.

The chemical action of the sky (*i.e.*, of the portion of it thus included) is seen to be half that of the Sun at 45° altitude; and at altitudes of the Sun below 13°, where little more than half the sky is included in each observation, to be greater than that of the Sun.

17. I found the chemical action of the Sun, exactly the same for the same altitude, at all seasons of the year and at all hours of the day, as far as the experiments went at Dacca, and I find in Somersetshire the same chemical action of the Sun at the same altitude as at Dacca. I have not been able to get exactly the same candle that I used at Dacca; and a difference in the composition of the stearine might possibly cause a small difference in the results, but I believe not one of much importance.

[The observations in Table J below in the postscript show that the difference is absolutely nil.—*27th October*, 1890.]

In the 'Phil. Trans.,' 1867, pp. 558—562, Roscoe says that for equal altitudes of the Sun the chemical intensities are equal; and he "assumes" that the same "relation between the Sun's altitude and chemical intensity holds good at Kew, Heidelberg, and Pará." These results of Roscoe are confirmed by my observations; he obtained them only by "averaging" numerous observations taken at Kew, and assuming that the effects of cloud, &c., in the long run were self-destructive.

Roscoe supposes that a marked difference which he found in intensity between spring and autumn might be due to a difference in transparency. I can only explain some of Roscoe's results by supposing that the sky was not perfectly clear at the time of the observations. Indeed, from the description, many of Roscoe's observations would appear to have measured the effects of cloudiness rather than of Sun and sky. I have no anomalies in the results of my observation except such as I think I may fairly attribute to cloud or haze. My experience in England is that it requires months of watching to catch a sky that will give results similar to those I obtained regularly in Dacca during the cold season.

In the 'Phil. Trans.,' 1867, p. 559, Roscoe finds (by the same method of "averaging") that "the relation between the Sun's altitude and the chemical intensity of total daylight is graphically represented by a right line." And in the 'Phil. Trans.,' 1870, p. 315, Roscoe and Thorpe say that the relation between altitude and total chemical intensity, for altitudes above 10°, is seen to be accurately represented by a straight line.

Table B indicates, and Table G below proves, that the straight line is only a first approximation to the truth. The calculation from my Table B of the chemical action of the whole visible sky (and Sun) on the horizontal plane can be effected, as shown further on in the present paper.

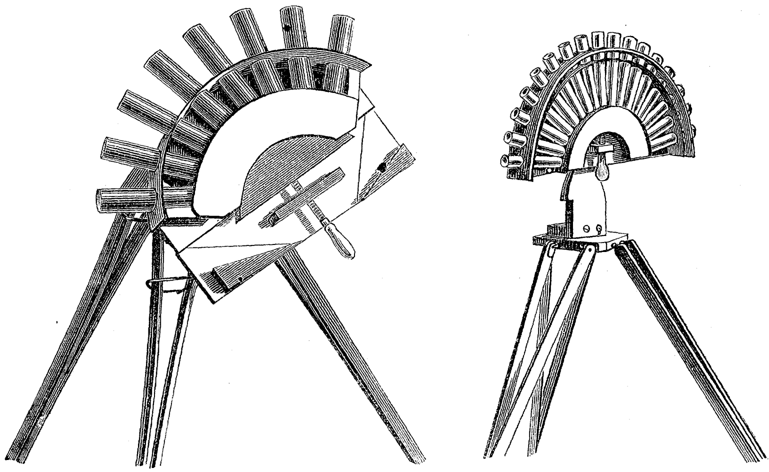
18. Various observations had led me to expect that the chemical action of the sky at the same moment was different in different parts of it. To investigate this suspicion, I designed an instrument which I call the *Mitrailleuse Actinometer* (fig. 2); I place in the President's hand photographs of two of these instruments.

I mount a number of similar cylindric tubes in one plane in a semicircle, to the centre of which each tube is directed. One extremity of each tube lies on the circumference of the circle; the other extremities lie on a concentric circle of about half the radius. In the circumference of this smaller circle, is a semicircular series of holes, against which a semicircular block, carrying the sensitised slip of paper, is pressed by a screw. Each cylinder in the first Dacca mitrailleuse cut out of the sky a circle of 8° 28′ angular diameter. One of the tubes near its top, carries a small plate of wood, on which stands a style parallel to the tube, by means of which the particular tube can be brought into a line with the Sun. By another motion the plane of all the tubes can be adjusted to the plane of symmetry (or elsewhere).

[A vertical plane through the Sun at any time divides the visible sky into two exactly similar portions. I will call it the plane of symmetry].

19. The observations (Table C) were taken 23rd December, 1864,

Fig. 2.



at Dacca (among many other similar observations taken in the same cold weather), in the plane of symmetry. The barrels of the mitrailleuse were fixed 10° apart, the altitude of the Sun being 42° 28′.

Table C.

Altitude of the axis of the barrel of the mitrailleuse.	Distance of axis of barrel from the sun $= \theta$.	Observed chemical action during six minutes' exposure $= i_\theta$.	Calculated value of i_θ from $i_\theta = 0{\cdot}12 \operatorname{cosec} \theta$.
10°	−32° 58′	0·2	0·2205
20	−22 58	0·6	0·3075
30	−12 58	0·7	0·5348
40	− 2 58	..	2·3186
50	+ 7 2	0·844	0·98
60	17 2	0·322	0·4097
70	27 2	0·188	0·264
80	37 2	0·184	0·1992
90	47 2	0·177	0·164
100	57 2	0·144	0·143
110	67 2	0·14	0·1304
120	77 2	0·128	0·123
130	87 2	0·122	0·1201
140	97 2	0·12	0·1209
150	107 2	0·128	0·1255
160	117 2	0·136	0·1347
170	127 2	0·136	0·1563

The readings of the chemical action are taken in terms of the unit of candle power, and were compared also with a graded Sun-strip, made at the same time from the same photographic paper by the water-motion actinometer, fig. 1.

The observations given by the barrels at 170° and 10° are too low, doubtless owing to haze so near the horizon. No observation could be made with the barrel at 40°, because the Sun could not be kept out of it. The observation made by the barrel at 20° is (apart from comparison with computed value) evidently erroneously large. I give the table as an early observation that shows well that there is a point of minimum sky intensity at 90° from the Sun. It also appears that if i_α represent this intensity for the altitude α of the Sun ($= 0{\cdot}12$), then the intensity of the sky at a point θ from the Sun is given (roughly only according to this table) by the formula

$$i_\alpha \operatorname{cosec} \theta.$$

This observation was made in the plane of symmetry: it turns out that the value, $i_\alpha \operatorname{cosec} \theta$, gives the intensity very accurately, in whatever plane θ be measured from the Sun.

I would note once more that my observations are largely comparative, and the results obtained are independent of the unit: it is not necessary to reduce the readings in this table to the one-second unit.

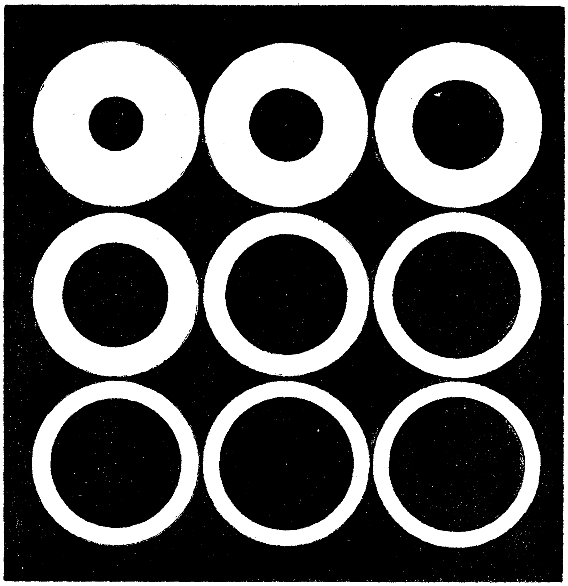
20. For any altitude of the Sun (α), the chemical action of the sky is a minimum at all points of a great circle 90° from the Sun, the plane of which is the plane of minimum intensity (i_α). And at this moment, the chemical action of the sky at any point distant θ from the Sun is given with great accuracy by the formula

$$i_\alpha \operatorname{cosec} \theta.$$

As the whole of the mathematical developments of this paper are founded on this law, I have been careful not only to verify it by numerous observations both at Dacca and in Somersetshire, but also to vary the form of the observations in every way I could devise.

21. Thus, the mitrailleuse has been placed in the plane of minimum intensity: in this case, all the barrels give accurately the same reading, except that those barrels 10° from the horizon read rather lower, as I anticipated they would; there must nearly always be *some* haze near the horizon.

Next, the mitrailleuse was placed at various angles with the plane of symmetry, by turning it round the line joining one of its tubes with the Sun. The observed chemical actions agree well with $i_\alpha \operatorname{cosec} \theta$. Next, by means of stops, I made the aperture of each barrel of a mitrailleuse to be $c \sin \theta$, where θ is the distance of the axis of the barrel from the Sun. This mitrailleuse being exposed,



the barrel with aperture $c \sin o$ being directed to the Sun, the circular darkened spots were found to be very accurately of uniform depth. Further, I calculated the times of exposure, for a (particular) mitrailleuse which ought, on the law $i_\alpha \operatorname{cosec} \theta$, to give a uniform tint. I exposed this mitrailleuse for these calculated times, first in the plane of symmetry, afterwards in a plane inclined to it at an angle of 52°; the results agreed closely with my anticipation, and show $i_\alpha \operatorname{cosec} \theta$ to be a very good approximation.

22. I have therefore made full use of the expression $i_\alpha \operatorname{cosec} \theta$ for the chemical action of the light of the sky in a circle distant θ from the Sun (whose altitude is α).

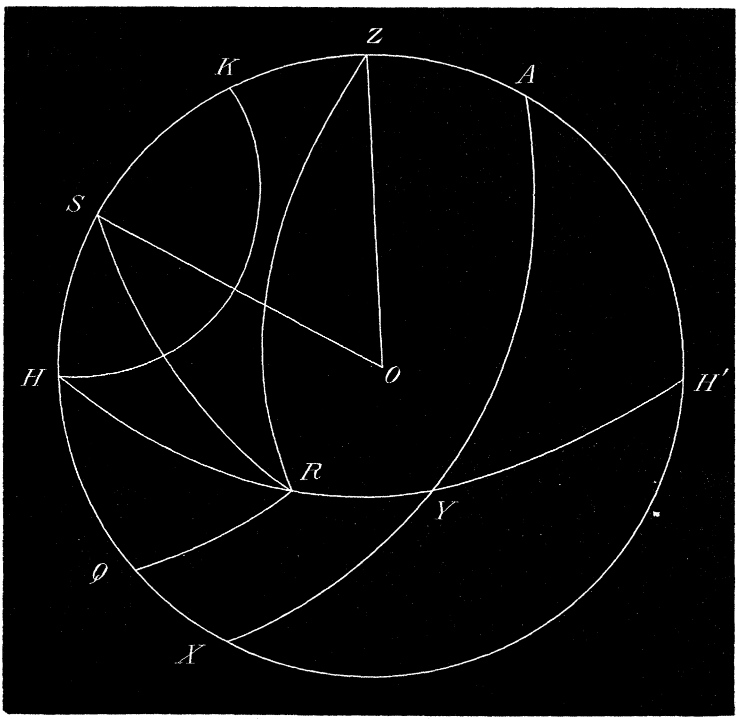
In carrying out integrations which include the portion of the sky actually occupied by the Sun, we do not, by employing this formula, introduce any infinite expression; for each circular band of the sky of small breadth $\delta\theta$ distant θ from the Sun has an area $2\pi \sin \theta \, d\theta$; the chemical action of such band is therefore $2\pi \, i_\alpha \, d\theta$: so that the total chemical action thus attributed to the sky in the area occupied by the Sun's disk would be inappreciable.

23. Bunsen and Roscoe ('Phil. Trans.,' 1859, p. 891) determined chemically the action of the rays falling from a measured portion of cloudless sky situated near the zenith, and then compared the *visual* luminosity of this same portion of zenith sky with that of the total heavens. They say "the amount of light chemically measured,

which falls from the same surface of zenith sky, multiplied by the preceding ratio, must give the chemical action which the whole sky would produce on a horizontal unit of surface."

I have below in one or two points only attempted to institute a numerical comparison between the results of Sir H. Roscoe and my own; considering the great difference in our methods, I am not surprised that no good coincidence in the results can be established.

DIAGRAM 1.



24. Having given i_α the chemical action in the circle of minimum intensity, to calculate the total chemical action of the sky on a plane exposed at right angles to the Sun.

(N.B.—i_α is a constant for this calculation, but it varies with α the altitude of the sun).

Let the figure (Dia. 1) represent a projection on the plane of symmetry, S being the Sun, Z the zenith, HRYH′ the horizon, AYX the plane of minimum intensity, SH $= \alpha$ the Sun's altitude.

Let θ be the angular distance from the Sun of the elementary zone

QR, and ϕ the angular distance of an element in the zone QR from SQ.

Denote by I_s the chemical action exerted by a circular area s of the sphere, on the plane at right angles to the Sun.

The area of an element will be $d\phi\, d\theta \sin\theta$, the intensity of the chemical action will be $i_\alpha \operatorname{cosec}\theta$.

The angle between the normal to the element considered and that to the plane AYX is θ.

$$\therefore d\,.\,I_s = d\phi\, d\theta \sin\theta \times i_\alpha \operatorname{cosec}\theta \times \cos\theta$$

$$\therefore I_s = i_\alpha \int_0^{2\pi}\int_0^{\theta} d\phi\,.\,d\theta\,.\cos\theta = 2\pi i_\alpha \sin\theta.$$

Or, for the whole hemisphere, of which the Sun is the pole,

$$I_H = 2\pi i_\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (Q),$$

from which, to get our desired result, we have to subtract the chemical action I_s of the gore XYH.

$$I_G = i_\alpha \int_\alpha^{\frac{\pi}{2}}\int_{-RSH}^{RSH} \cos\theta\,.\,d\theta\,.\,d\phi = 2i_\alpha \int_\alpha^{\frac{\pi}{2}} RSH \cos\theta\,.\,d\theta.$$

$$(\cos RSH = \tan SH \cot SR = \tan\alpha \cot\theta).$$

$$\therefore I_G = 2i_\alpha \int_\alpha^{\frac{\pi}{2}} \cos^{-1}(\tan\alpha\cot\theta)\cos\theta\, d\theta$$

$$= 2i_\alpha \left[\operatorname{Lim}_{\alpha}^{\frac{\pi}{2}} \{\cos^{-1}(\tan\alpha\cot\theta)\sin\theta\} - \int_\alpha^{\frac{\pi}{2}} \frac{\tan\alpha\,.\,d\theta}{\sqrt{(1-\sec^2\alpha\cos^2\theta)}}\right]$$

$$= 2i_\alpha \left[\frac{\pi}{2} - \int_\alpha^{\frac{\pi}{2}} \frac{\tan\alpha\,.\,d\theta}{\sqrt{(1-\sec^2\alpha\cos^2\theta)}}\right].$$

Whence, subtracting this from equation (Q),

$$I_H - I_G = i_\alpha \left\{\pi + 2\int_\alpha^{\frac{\pi}{2}} \frac{\tan\alpha\,.\,d\theta}{\sqrt{(1-\sec^2\alpha\cos^2\theta)}}\right\}.$$

This expression cannot be integrated in finite terms, but, by using a formula of reduction in series, it gives

$$I_H - I_G$$
$$= i_\alpha \left\{\pi + 2\pi \frac{\sin\alpha}{1+\sin\alpha}\left[1 + 0{\cdot}25\tan^4\left(\frac{\pi}{4}-\frac{\alpha}{2}\right) + 0{\cdot}14\tan^8\left(\frac{\pi}{4}-\frac{\alpha}{2}+\right)\cdots\right]\right\},$$

which is the formula I have used in numerical computations. $I_H - I_G$ is the numerical value in the column headed "Sky alone" in

Table B, which is thus brought into direct verification with i_α, observed by the mitrailleuse.

An example of the actual calculation of i_α is added in Appendix B, not for publication.

25. The values for $I_H - I_G$ for different altitudes of the Sun in Table B are much the most trustworthy observations, and are the means obtained from a very large number of observations. I have, therefore, by the formula obtained in the last paragraph (24), inversely calculated the value of i_α for every 5° within the limits 5° to 40°, and placed them in Table D.

Table D.

Sun's altitude.	i_α calculated from Table B (column headed "Sky alone").
5°	0·00329
10	0·00681
15	0·00928
20	0·01073
25	0·01144
30	0·01188
35	0·01205
40	0·01218
45	0·01213
50	0·01209
55	0·01204
60	0·01200
65	0·01195

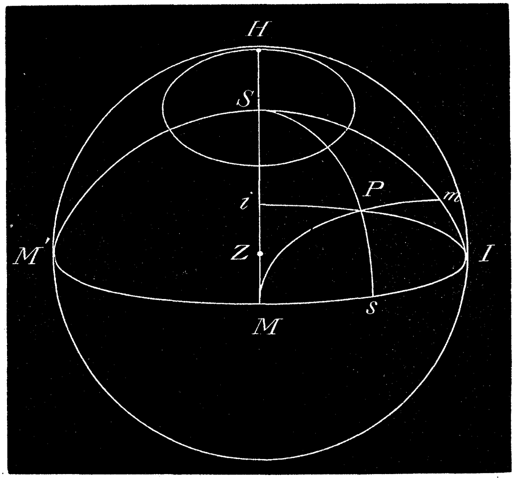
26. *Theorem.*—On the resolution of the chemical action of the sky in a direction perpendicular to any plane.

The figure (Dia. 2) is supposed an orthographic projection of the visible hemisphere on the plane of the horizon; S being the Sun, Z the zenith, HSZM the projection of the plane of symmetry, M′MI that of the plane of minimum intensity, and M′SI that of the plane through S at right angles to each of the other planes (which I call the plane of the Sun's altitude). These three planes, when produced, divide the sphere into eight quadrantal surfaces, of which SMI is one. In the quadrantal triangle SMI, S, M, I are the poles of the opposite sides.

Let the polar coordinates of P (an element of the surface) be PSZ $= \phi$ and SP $= \theta$. Then, as before, the element will have an area $d\phi.\ d\theta.\ \sin\theta = i_\alpha\ d\phi.\ d\theta.$

Let the planes OSM, OSI, and OIM (O being the centre of the hemisphere) be taken as coordinate planes; OS, OM, OI, the three axes

DIAGRAM 2.



of coordinates; and suppose through P the three quadrants to be drawn from S, M, I, to the opposite sides, meeting them in s, m, i respectively. Then the normal chemical action $i_\alpha\, d\phi.\, d\theta$ may be resolved in three directions parallel to SO, MO, IO; and the three components in these directions will be respectively $i_\alpha\, d\phi\, d\theta \sin \mathrm{P}s$, $i_\alpha\, d\phi\, d\theta \sin \mathrm{P}m$, $i_\alpha\, d\phi\, d\theta \sin \mathrm{P}s$. Call these respectively $d^2\mathrm{U}$, $d^2\mathrm{V}$, $d^2\mathrm{W}$.

We have

$$\mathrm{P}s = \frac{\pi}{2} - \theta, \qquad \sin \mathrm{P}m = \sin\theta \cos\phi, \qquad \sin \mathrm{P}i = \sin\theta \sin\phi,$$

and hence

$$\mathrm{U} = i_\alpha \iint d\phi\, d\theta \cos\theta, \qquad \mathrm{V} = i_\alpha \iint d\phi\, d\theta \sin\theta \cos\phi,$$

$$\mathrm{W} = i_\alpha \iint d\phi\, d\theta \sin\theta \sin\phi.$$

27. To find the value of these expressions for the hemisphere having the Sun at its apex; we have to take ϕ from $-\pi$ to π, and θ from 0 to $\frac{1}{2}\pi$, which gives

$$\mathrm{U} = 2i_\alpha \pi, \qquad \mathrm{V} = 4i_\alpha, \qquad \mathrm{W} = 0.$$

28. To find the chemical action of the hemisphere about S resolved on the horizontal plane Q_s, we have

$$d^2(\mathrm{Q}_s) = d^2\mathrm{U} \sin\alpha + d^2\mathrm{V} \cos\alpha$$

whence

$$Q_{(s)} = i_\alpha \sin\alpha \iint d\phi \, d\theta \cos\theta + i_\alpha \cos\alpha \iint d\phi \, d\theta \sin\theta \cos\phi$$

$$= 2i_\alpha (\pi \sin\alpha + 2\cos\alpha) \quad \text{(X)}.$$

29. This is a mere literal result: what is required is (Q_z); *i.e.*, the chemical action of the hemisphere about Z resolved on the horizontal plane; that is, the relation between the chemical action in the plane of minimum intensity i_α (Sun's altitude α) and the "total chemical action of diffused daylight" as observed by Roscoe on horizontally exposed paper.

The answer (Q_z) is identical in form with (Q_s) as given in equation (X) above; but the limits of θ are functions of ϕ which lead to elliptic integrals.

Referring back, however, to diagram 1, it will be seen that (Q_z) differs from (Q_s) by the addition of the gore AYH, the subtraction of the gore HYX; which will be found to be no difference at all; as the values of the chemical action of each element in the subtracted gore are equal to those for a corresponding element in the added gore, with the same sign and angles of resolution on the horizontal plane. Hence we must have—

$$(Q_z) = (Q_s) = 2i_\alpha (\pi \sin\alpha + 2\cos\alpha) \quad \text{(Y)}.$$

As this is a result of the first importance, I submit at the end of this paper in an Appendix, not for publication, the work by which I first arrived at the equation (Y) by laborious transformation of the elliptic integrals, which are reduced finally so that two terms, each irreducible by integration in algebraic form, destroy each other.

30. The results thus arrived at by employing the law of the cosecant are so neat that a suspicion may arise that the law may have been assumed as one lending itself to mathematic manipulations.

I may be permitted, therefore, to state, that the law was arrived at, more than twenty-two years ago, by experiment simply, and the subject soon after laid aside. The present mathematic investigations were only recommenced within the last two years, in order to institute a comparison between my old Dacca observations and those of Sir H. Roscoe.

31. In 'Phil. Trans.,' 1870, p. 314, Sir H. Roscoe gives a table showing the total chemical action of diffuse daylight (*i.e.*, of the whole sky, the Sun being stopped off) on horizontally exposed paper. These observations were taken in Portugal, with a perfectly clear sky, and I therefore select them for comparison with the foregoing theory and observed values of constants.

Columns 1 and 2 are copied from Roscoe, *l.c.*; column 2 gives my $Qz = 2l_\alpha\,(\pi \sin \alpha + 2 \cos \alpha)$. In column 3 I give l_α, calculated from this equation. In column 4 I place the values of i_α obtained from table B, the "sky alone" column, by the aid of the formula at the end of Art. 24.

In the 5th column the values in column 4 are brought up by proportion for comparison with those in column 3, taking the observation at altitude 42° 13′ as the best; *i.e.*, increasing all the numbers in the ratio of 121 to 160.

Table E.

1. Sun's altitude.	2. Diffuse daylight.	3. i_α calculated from 2.	4. i_α calculated from Table B.	5. Values in Column 4 brought up.
9° 51′	0·038	0·0078	0·0068	0·0090
19 41	0·062	0·0105	0·0107	0·0141
31 14	0·100	0·0150	0·0118	0·0156
42 13	0·115	0·0160	0·0121	0·0160
53 9	0·126	0·0170	0·0121	0·0160
61 8	0·132	0·0177	0·0120	0·0159
64 14	0·138	0·0187	0·0120	0·0159

The discrepancies do not appear at first sight great between the results of Sir H. Roscoe and my own. But his observations would show the maximum value of i_α attained when the Sun was at or near the zenith, mine that this maximum occurs when the Sun is about 45° or 50° altitude.

It is true that in the Dacca Table B, the actual observations extend only to 45° or thereabout, and that the values for altitudes of the Sun above 45° are only filled in hypothetically; but my best established observations at Dacca, for altitudes of the Sun from 30° up to 45°, show directly that at altitudes of the Sun of 45° or 50° the value of i_α would reach a maximum.

In my Dacca observations, each additional 5° to the Sun's altitude brings into effect an additional 5° gore of the sky. It is therefore clear (apart from the law $i_\theta = i_\alpha \operatorname{cosec} \theta$ and the integrations consequent thereon) that i_α will have a maximum value when α, the Sun's altitude, is about 50° or 60°.

I am not surprised that so considerable a discrepancy results from a comparison of the observations. In a single series of observations, the incidental errors of reading, &c., would introduce into the small numbers given in column 3 sufficient differences to alter entirely the law indicated for i_α.

32. Since in table E the value of i_a for $\alpha = 42° 13'$ is found from Roscoe's observations to be 0·016, from mine to be 0·012, it follows that Roscoe's unit of chemical action is $\frac{12}{16}$ of my Dacca candle unit. This is merely a first attempt to correlate these units.

33. The resultant chemical action of the sky on a horizontally exposed piece of paper, the Sun's altitude being α, is found

$$= (2\pi \sin \alpha + 4 \cos \alpha) i_\alpha.$$

This vanishes when

$$2\pi \sin \alpha + 4 \cos \alpha = 0,$$

i.e., when
$$\tan \alpha = -\frac{2}{\pi},$$

or
$$\alpha = -32° 29'.$$

This gives an absolute value for twilight, supposing daylight to cease when the diffused daylight of Roscoe entirely vanishes.

The extreme limit at which twilight has been certainly observed is when the Sun was 24° below the horizon; at which time the formula $i_\alpha(2\pi \sin \alpha + 4 \cos \alpha)$ would show the chemical action of diffuse daylight to be only $\frac{1}{40}$ part of what it was just after sunset.

In other words, the formula

$$i_\alpha(2\pi \sin \alpha + 4 \cos \alpha)$$

gives a very good agreement with the observed duration of twilight, supposing, that is, the illumination and the chemical action to follow much the same laws in this extreme case.

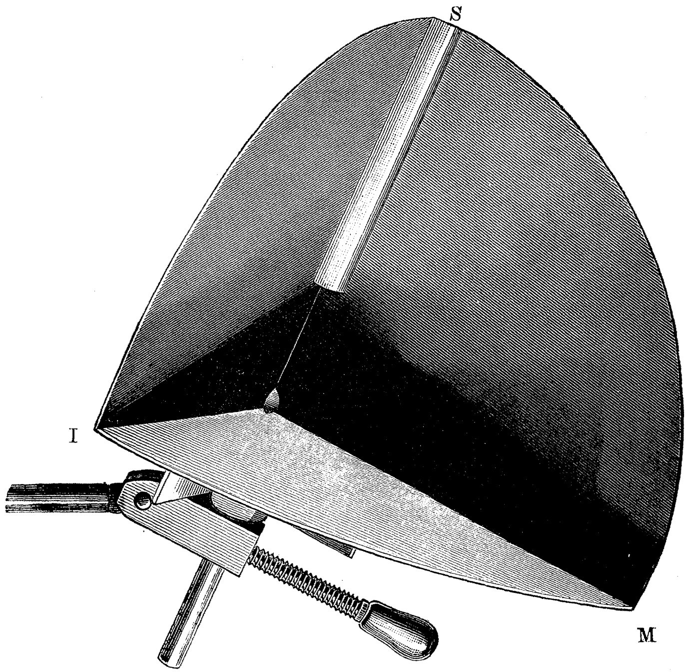
34. Taking up the expressions for U, V, W at the end of Art. 26, I integrate them for the octant of the sphere contained by the three coordinate planes, viz., the plane of symmetry, the plane of minimum intensity, and the plane of the Sun's altitude; *i.e.*, I take ϕ and θ each from 0 to $\frac{1}{2}\pi$; which gives

$$[\mathrm{U}] = \frac{\pi}{2} i_\alpha, \qquad [\mathrm{V}] = [\mathrm{W}] = i_\alpha.$$

This suggested the construction of the octant actinometer, which requires only one-fourth of the visible sky to be clear for observation, and gives the value of i_α directly, requiring no calculations of reduction.

35. The *octant actinometer* (fig. 3) consists of three quadrantal planes, MOS, MOI, and IOS, joined at their edges so as to form a hollow trihedral, and mounted so that one of the edges, OS, can be brought to point to the Sun, and the plane MOI will then coincide with the plane of minimum intensity. The instrument has another adjustment

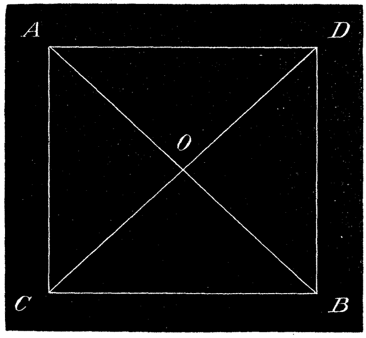
Fig. 3.



by which it can turn round OS as an axis, and if one of the planes MOS, IOS be brought to coincide with the plane of symmetry, the other will coincide with the plane of the Sun's altitude.

I take a small square (diagram 3) of sensitised paper and cut it along CO ; then, slipping the part COB under AOC, so that B coin-

Diagram 3.



cides with C, it forms a rectangular trihedral of sensitised paper. This is placed in a small exposure trihedral of cardboard, and covered by a thin metallic trihedral in the trihedral angle of the octant. (I make several of these trihedrals of sensitised paper, so as "in the field" to take quickly a series of observations.)

The trihedral of sensitised paper is, of course, carefully covered up till the instrument is in adjustment; if then exposed to the action of the sky for (say) 30 seconds, the readings on the quadrantal planes MOS and IOS will be each 30 i_a, and that on the quadrantal plane MOI will be 30 . $\frac{1}{2}\pi$. i_a.

36. I tried this octant actinometer on the 13th August, 1890—the first day that the sky had been partially clear for a long time—and also (with a more imperfect sky) on the 15th and 16th August, 1890, at Milverton, near Taunton. The exposures were all for 30 seconds. I give the whole results.

Table F.

Time.	Sun's altitude.	[V] = [W]. i on the two planes.	[U]. i on the third plane.
1890. 12th Aug., 5.0 P.M. ...	21° 30′	0·0187	0·024
5.10 ...	19 30	0·0183	0·025
5.20 ...	18 0	0·0191	0·027
5.33 ...	16 15	0·0191	0·023
5.42 ...	14 30	0·0183	0·027
5.47 ...	13 30	0·0150	0·023
15th Aug., 11.25 A.M. ...	52 30	0·0270	0·030
11.30 ...	53 0	0·0240	0·027
16th Aug., 0.45 P.M. ...	52 15	0·0170	0·019
0.50 ...	52 0	0·0190	0·028
4.0 ...	29 0	0·0170	0·020
4.15 ...	27 30	0·0150	0·019

It is evident that these observations were interfered with greatly by haze or cloud; but it may be well to explain exactly how they were taken.

A "sunstrip" was shaded first by the water-motion actinometer; the altitude of the Sun being known, the value of any line in this sunstrip, in terms of the Dacca candle unit, was known by the aid of Table B.

The adjustment and working of the octant actinometer were found not difficult. The readings in the two planes [V] and [W] were found practically equal; the results are in the third column. These "readings" were obtained directly by comparison with the "sun-

strip," and divided by 30 are the numbers in column 3. Similarly, the numbers in Column 4 represent [U].

$$\text{Now } [V] = [W] \text{ should be } \frac{2}{\pi} \cdot [U].$$

These observations do not give [U] large enough.

Also, the observations of 12th August would show the value of i_α when $\alpha = 20$ to be about 0·018 or 0·019. But the Table D shows the true value of i_α when $\alpha = 20°$ to be 0·0107; that is to say, the readings of 12th August, 1890, with the octant actinometer are altogether too high. This may easily be so without any fault in the instrument or error in the observations, and on two reasons. First, the presence of any bright cloud may have given the readings [V] and [W] too large. (Bunsen and Roscoe, in 'Phil. Trans.,' 1859, p. 905:—"These observations prove that the presence of a thin film of cloud increases the amount of chemical illuminating effect in the most striking manner."—The clouds "act as mighty reflectors of light.") Secondly, a very slight haze over the Sun would give the sunstrip too low, and thus largely increase the results of columns (3) and (4) read by it.

I do not consider these observations to decide anything as to the merits of the octant actinometer, which can only be satisfactorily tested by the sky of Dacca or some similar subtropical or tropical station.

37. It is difficult to determine which method of resolution of the sky and Sun gives the most useful measure of the general total effect, whether for determining the time of exposure of a photographic plate or for estimating the effect on vegetation. Sir H. Roscoe has taken (for the sky) the resultant action on paper exposed horizontally; I append, therefore, in Table G the chemical action similarly measured, so that column 2 is exactly = the "diffuse daylight" of Roscoe, and column 4 = the "total daylight" of Roscoe. This table is deduced by calculation from the Dacca Table B, by the aid of the law $i_\theta = i_\alpha \operatorname{cosec} \theta$, *i.e.*, from the value for $I_H - I_G$ in Art. 24, and the value $Q_z = 2i_\alpha (\pi \sin \alpha + 2 \cos \alpha)$, which are directly derived from that law.

This table, as far as $\alpha = 45°$, is a direct consequence of the Dacca observations. The values given from 50° up to 60° are a theoretical extension, perhaps as near as would be given by interpolation between known extremes. I do not think the numbers for α from 60° to 90°, which might be arrived at in a similar way, would have any real value.

This table, equally with the Dacca Table B, shows how large the sky effect is in comparison with the Sun effect, especially for altitudes of the Sun below 30°. This may be the explanation of the reason why trees close to the *north* side of greenhouses exercise a prejudicial influence.

Table G.—Showing Chemical Action of the Sun and of the whole Sky, resolved on the Horizontal Plane, for various Altitudes of the Sun (the Sky being perfectly clear from Cloud and Haze).

1. Sun's altitude.	2. "Diffuse daylight" of Roscoe.	3. Sun × sine altitude.	4. Sum of columns 2 and 3.
5°	0·0150	0·0006	0·0156
10	0·0343	0·0041	0·0384
15	0·0510	0·0122	0·0632
20	0·0625	0·0240	0·0865
25	0·0718	0·0382	0·1100
30	0·0784	0·0535	0·1319
35	0·0830	0·0697	0·1527
40	0·0865	0·0856	0·1721
45	0·0882	0·1010	0·1892
50	0·0893	0·1149	0·2042
55	0·0900	0·1285	0·2181
60	0·0893	0·1403	0·2296

The present paper contains my Dacca experiments and numbers arrived at by calculation therefrom. I have been for a year making similar experiments in England whenever the sky by its clearness offered any chance of a good observation; but I have not been able to get any observation such that I should attempt to correct the Dacca Table B thereby. I, therefore, am satisfied to publish the present paper in its present form, leaving to others its extension by the help of further observations under a perfectly clear sky.

38. *Postscript*, 15*th October*, 1890.—I have within the last few days made a number of observations with the octant actinometer, and have also, by making a few sunstrips at different altitudes, compared the times for the candle unit with those of the Dacca tables. These observations, though giving no numerically valuable results, strongly confirm the views I have expressed in this paper, and I append a statement of them.

On the 10th October the sky was seemingly clear; but, the values obtained for [V] = [W] being much too high, I did not continue the observations.

On the 11th October I took a sunstrip at $12^{h}\ 8^{m}$, the Sun's altitude being 31° 30′; comparing this afterwards with a candlestrip, I found the time for the candle unit to be 8·5 seconds. Referring to the Dacca table, I found the time for the same altitude, 31° 30′, to be 8·9 seconds. I therefore used this sunstrip for the observations in the preceding table. I infer that, at least at $12^{h}\ 8^{m}$, the sky on the 11th October was really clear. Some of the values in this table are higher than those obtained by computation for i_{α}.

Table H.—Octant Observations at Milverton, Somerset, 1890.

Time.	Sun's altitude.	[V] = [W].	[U].
10th Oct., 11h 15m	31° 0′	0·0237	0·C293
12 19	31 45	0·0225	0·0270
2 11	25 10	0·0130	0·0171
11th Oct., 12 10	31 15	0·0226	0·0312
1 8	29 30	0·0150	0·(250
1 44	27 30	0·0120	0·0298
2 13	24 20	0·0120	0·0208
3 9	19 0	0·0104	0·0208
3 43	14 30	0·0094	0·0156
4 22	8 45	0·0052	0·0073
5 9	Sunset.	0·0013	0·0033

On the 13th October, 1890, I made a series of octant observations; but, as I doubted whether the sky was really clear (*i.e.*, as the clear sky of Dacca in the cold weather), I made a series of sunstrips, as under:—

Table J.

Time.	Sun's altitude.	Exposure for candle unit (Milverton).	Exposure for same (Dacca).
1890. 13th Oct., 11h 35m ...	30° 15′	9·0 secs.	9·25 secs.
12 0 ...	30 30	9·2	9·2
12 33 ...	29 45	9·4	9·4
1 5 ...	28 45	0·75	9·75
1 35 ...	27 45	16·0	10·0
2 13 ...	23 30	14·75	12·0
2 54 ...	19 15	19·0	15·0

In the first four observations, the sky was apparently, and doubtless really, clear; in the three latter observations, some slight invisible cloud over the Sun produced great changes in the sunstrips.

From the exact coincidence in the readings in the four first observations, at Dacca and Milverton, I think it follows (1) that there was no material difference in my candles at Dacca and Milverton; (2) that the chemical action of the Sun at the same altitudes was the same at Dacca and Milverton.

It is also clear that the number of really fine hours of sky in England (*i.e.*, when it can be compared with the Dacca cold-weather sky) is very small—perhaps not a score in the year. And further that, in a great many apparently clear skies in England, there is

present some haze, visible or invisible, that affects the readings of the chemical action on sensitised paper very largely, even to 50 per cent.

The octant observations, the intensities estimated on the *first* of the above strips, are as follows :—

Table K.

Time.	Sun's altitude.	[V] = [W].
1890. 13th Oct., 12^h $4\frac{1}{2}^m$	30° 45′	0·0268
1 $8\frac{1}{2}$	28 30	0·0267
1 39	27 30	0·0251
2 $18\frac{1}{2}$	23 15	0·0244
2 59	18 30	0·0216
3 24	15 45	0·0194
4 $11\frac{1}{2}$	9 45	0·0055
4 48	3 0	0·0041

The values of [V] = [W], being so much greater than I expected, led me to imagine that, though the sky was apparently clear, the observations might have been affected by the hygrometric state of the atmosphere. There had been a fog in the morning, and the air was, though translucent, saturated all the afternoon.

The next day, the 14th October, was a similar day (fog in the morning), and I commenced octant observations much earlier in the morning. The results are given in the following table :—

Table L.—Octant Observations, 14th October, 1890.

State of sky.	Time.	Sun's altitude.	[V] = [W].	[U].
Clear sky, but slight fog faintly to be seen.	9^h 38^m A.M.	22° 48′	0·0217	0·0217
	10 8	25 15	0·0235	0·02[illegible]4
	10 39	27 30	0·0272	0·0272
	11 2	29 0	0·0217	0·0326
	11 32	29 45	0·0217	0·0326
	12 1 P.M.	30 45	0·0272	0·0399
	12 30	30 0	0·0290	0·029
Light fleecy clouds in the sky octant.	1 0	29 0	0·2540	0·3990
	1 30	27 30	0·3080	0·0435
Clear, but still faint clouds.	2 19	24 0	0·2720	0·2720
Clear.............	2 45	21 0	0·0210	0·3290
Partial clouds	3 15	17 15	0·0163	0·0355
	4 0	11 30	0 0091	0·0127

The effect of the faint fog in increasing the value of [V] = [W] is plainly seen in the morning observations. The effect also of a very few faint fleecy clouds is seen in the increase of [V] and of [U] for the observations at $1^h\ 0^m$ and $1^h\ 30^m$, before which no clouds had been visible. The air was saturated the whole day.

The candles which I used in all these observations, were the "Belmont Sperm," supplied to me so as to burn 100 grs. in 47 minutes.

"On the Minute Structure of the Muscle-Columns or Sarcostyles which form the Wing-Muscles of Insects. Preliminary Note." By E. A. Schäfer, F.R.S. Received December 15, 1890,—Read January 8, 1891.

[Plates 4 & 5.]

The fibres of the wing-muscles of most insects are made up of readily separable longitudinal elements, which are often called the "wing-fibrils," although several observers have remarked the existence of an apparently fine fibrillation in them. To avoid ambiguity, I shall employ the term "muscle-columns" (*Muskel-säulchen*, Kölliker), or its equivalent "sarcostyles,"* to designate these elements. They are united together to form the fibres by a not inconsiderable amount of granular interstitial substance (*sarcoplasm*, Rollett). This substance has been regarded (Ramón y Cajal) as the true contractile material of the muscles, but it is easy, nevertheless, to observe the contraction of the sarcostyles, isolated in white of egg, a fact which has been pointed out by more than one writer on the subject (Merkel, Kölliker).

If an insect of which the wing-muscles are of the character above described is cut open and placed in alcohol of about 90 p.c. for twenty-four hours or more, and is afterwards transferred to glycerine, the sarcostyles of the wing-muscles can be isolated and examined without difficulty; they exhibit almost every phase of extension and retraction (or contraction), and the usual appearance of alternate dark and light transverse bands, with a fine line traversing each light band. When stained with dyes, such as hæmatoxylin, the dark bands are found to take the staining most intensely; the fine transverse lines are much less stained, and the clear bands hardly at all. The various parts of the sarcostyle evidently differ from one another in their behaviour to staining reagents, and the transverse striation is not to be explained by the effect of the varicosities of the sarcostyle upon the light transmitted

* Σάρξ, flesh, στύλος, a column.

FIG. 1.

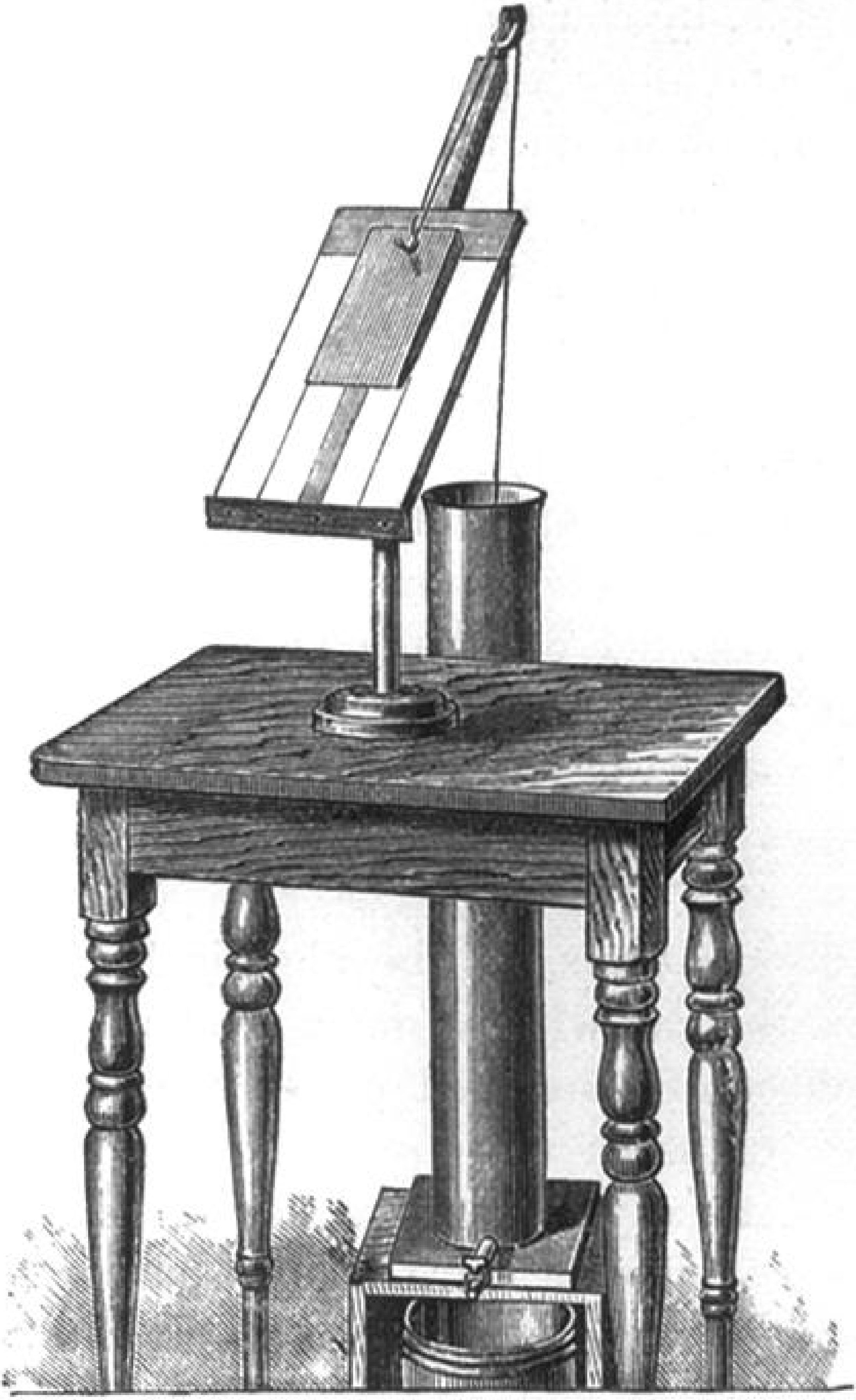
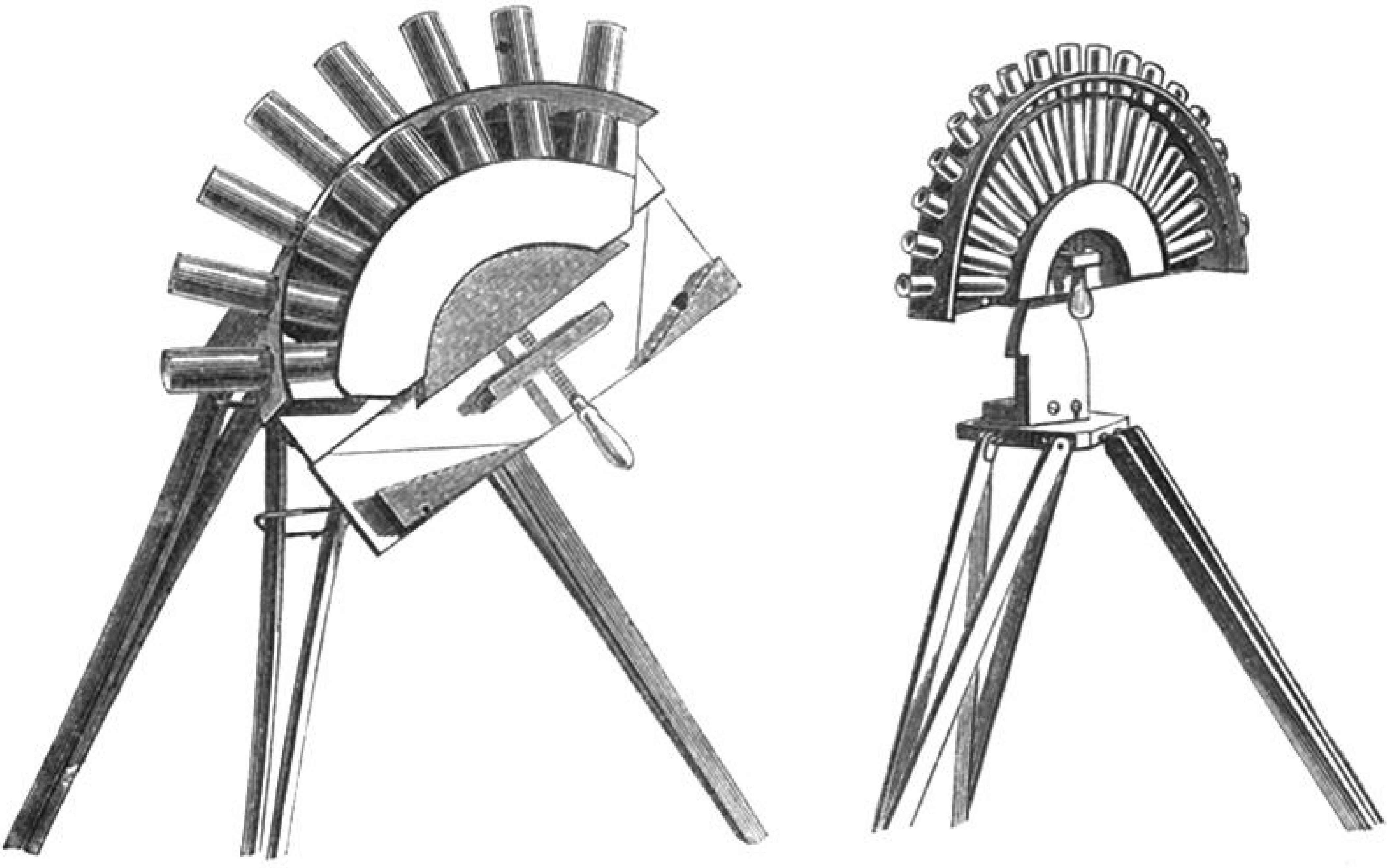


FIG. 2.



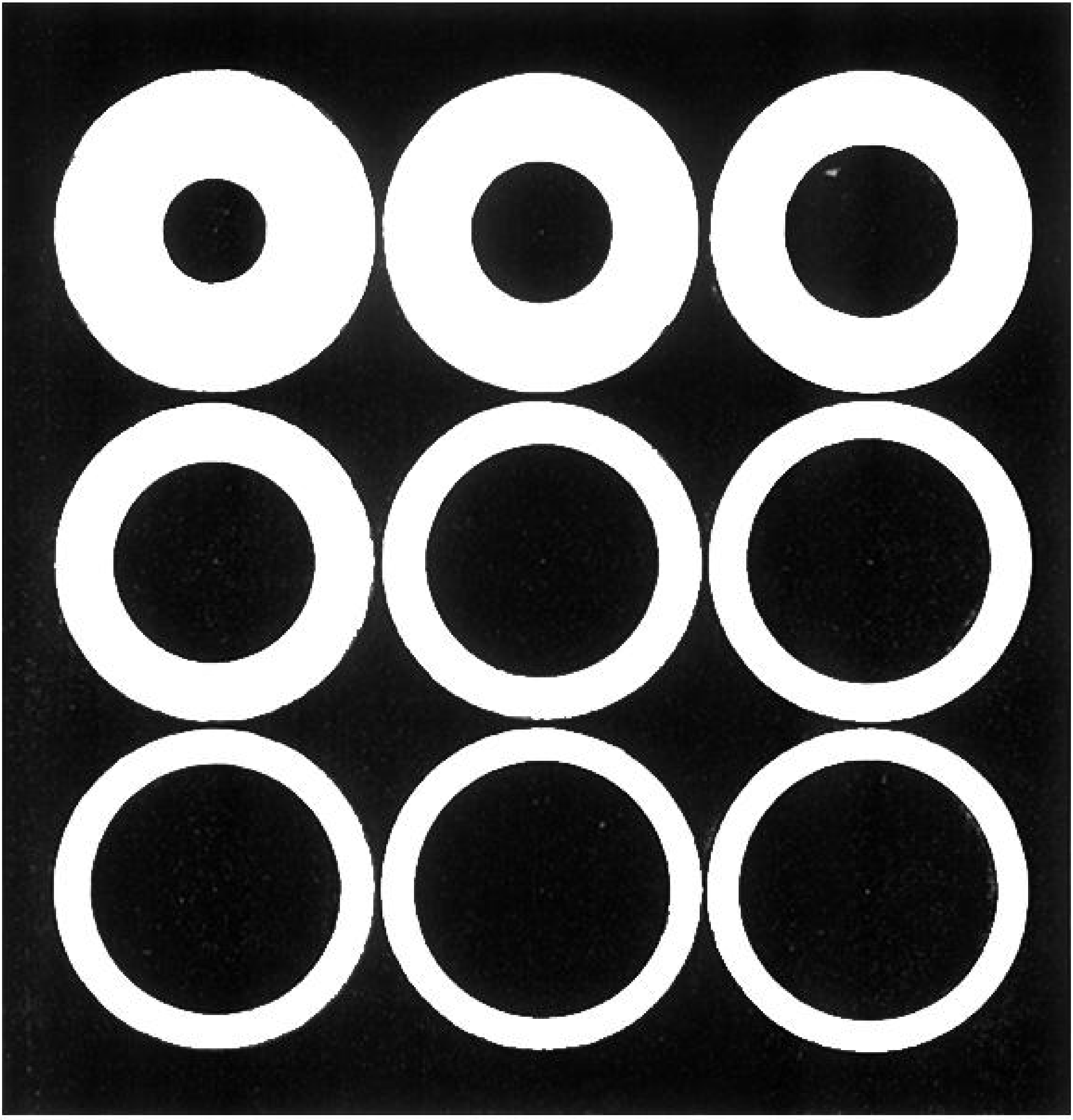


DIAGRAM 1.

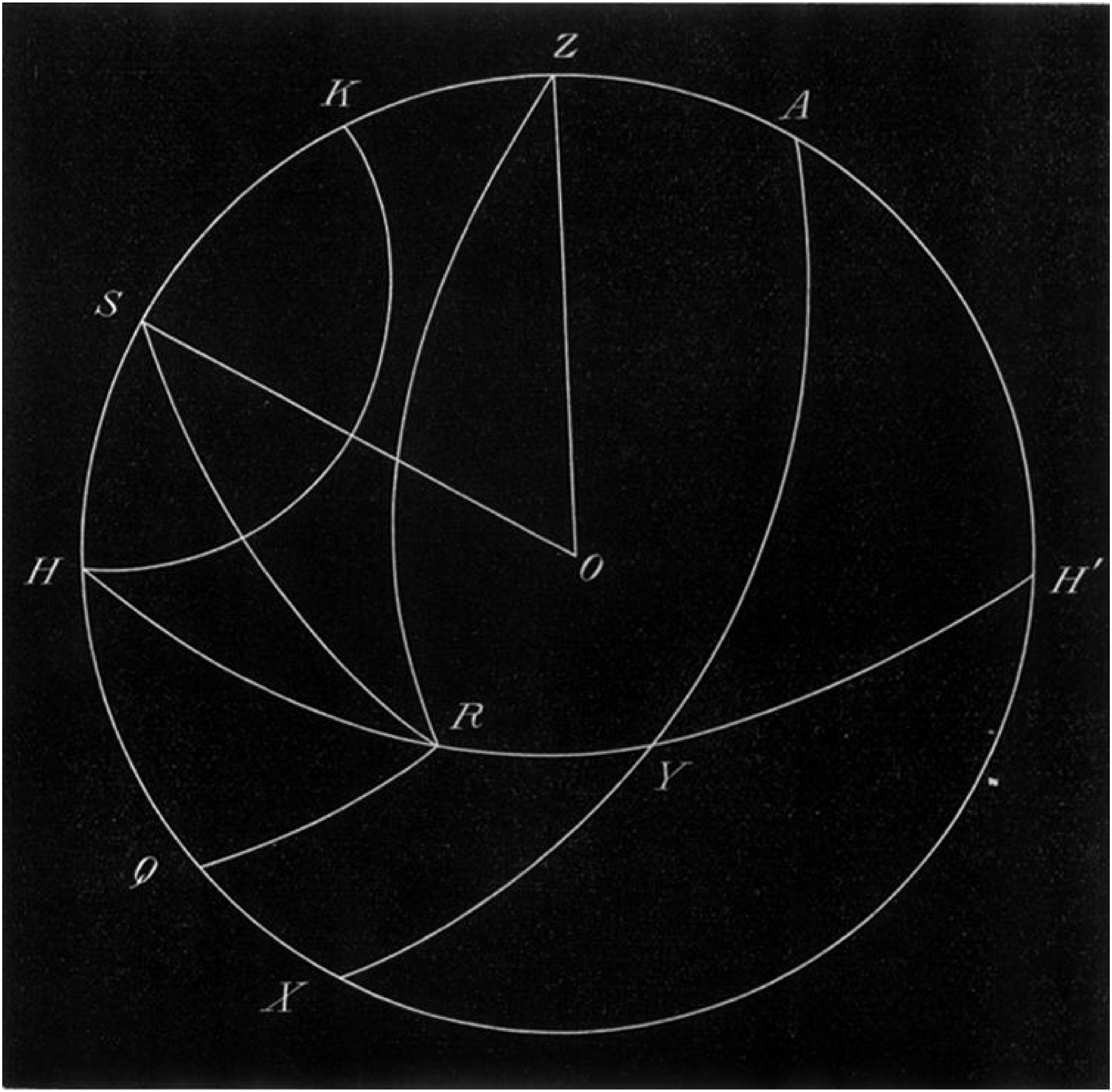


DIAGRAM 2.

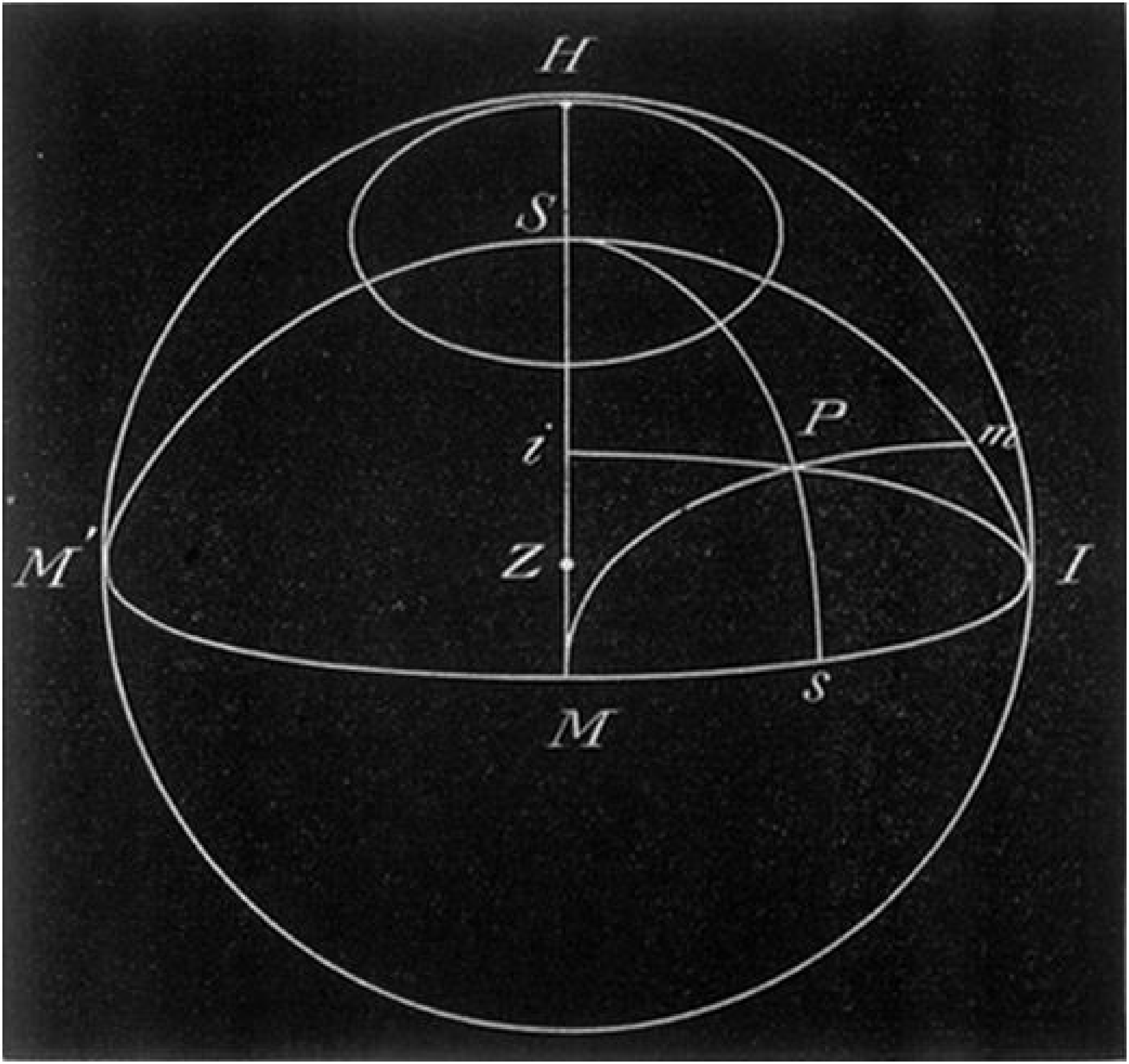


FIG. 3.

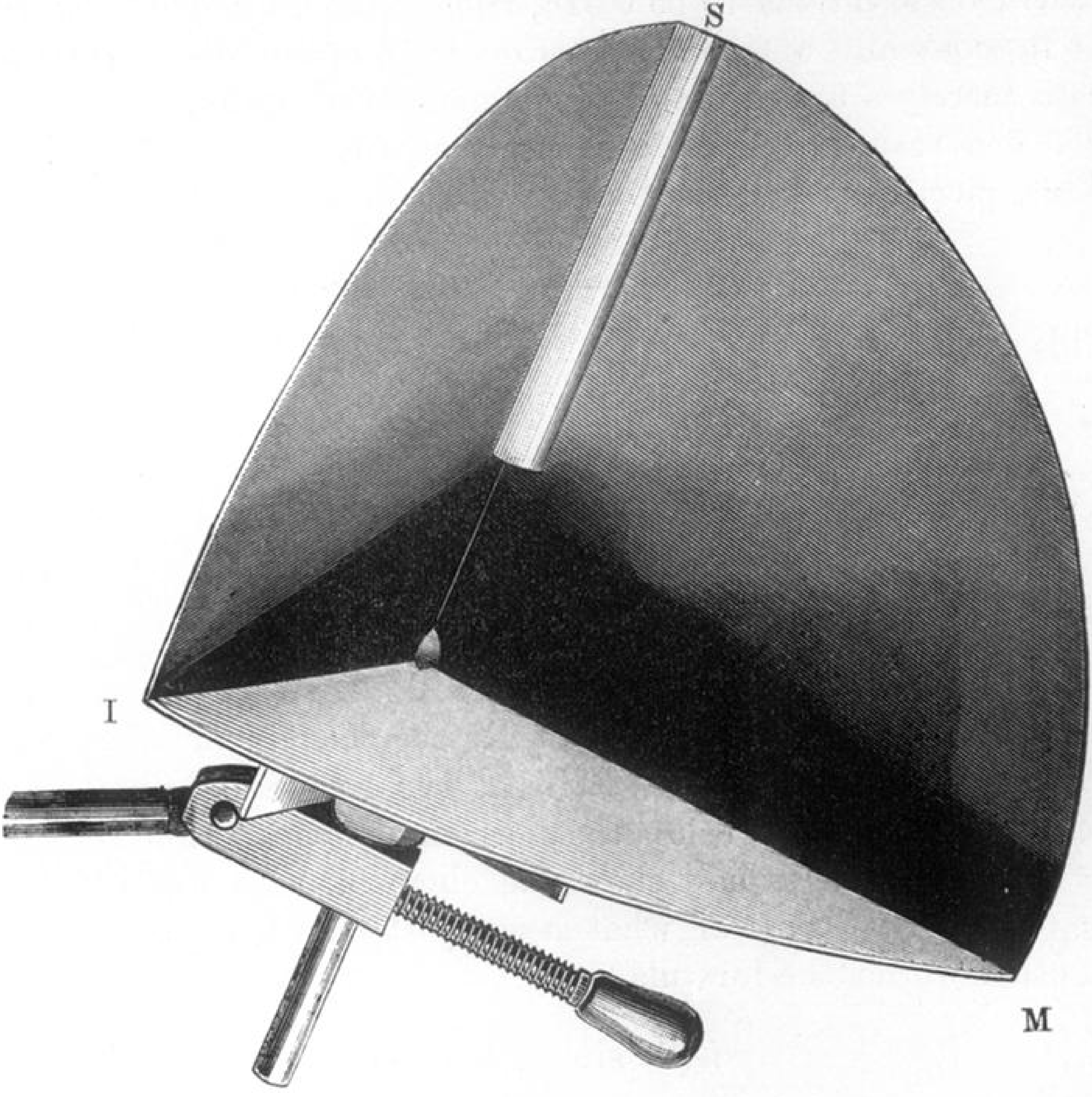


Diagram 3.